MATH REVIEW PROBLEMS   XY - KEY

Answers:

6) D   7) B   8) C   9) B   10) B   11) B   12) E   13) A   14) B   15) A

16) C   17) D   18) B   19) B   20) C   21) A   22) C   23) E   24) D   25) E

Solutions:

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6)   Answer: D   **Subject Area Tested: Inequalities**

This problem tests the students ability to solve inequalities. To solve, we must first get x by itself on one side of the equation.

$$-4x + 8 \leq 20$$
$$-4x \leq 12$$
$$x \geq -3, \text{ Choice D}$$

Remember, when you multiply or divide each side of an inequality by a negative number, you must reverse the direction of the inequality sign.

7)   Answer: B   **Subject Area Tested: Fractions**

First, you must find a common denominator for the 4 fractions which measure the contents of the 4 tanks: $2/3$, $7/10$, $4/5$, and $14/15$. The least common denominator is 30. So, now we must convert these four fractions to 30ths: $2/3 = 20/30$, $7/10 = 21/30$, $4/5 = 24/30$, and $14/15 = 28/30$.

With these re-expressed fractions, we can quickly solve the problem:

If you pour from the 4th tank ($28/30$'s full), you will pour $10/30$ into tank 1, $9/30$ into tank 2, and $6/30$ into tank 3. You have $3/30$ remaining in bottle 4 (28 - 10 - 9 - 6 = 3). $3/30 = 1/10 = 10\%$, Choice B.

8)   Answer: C   **Subject Area Tested: Percentages**

First, let's find the percentage of the track that will be made of concrete:

$$100\% - 44\% \text{ asphalt} - 36\% \text{ gravel} = 20\% \text{ concrete}$$

Next, we must calculate the square yard specifications for the concrete. We are told that the track is 200 square yards. So, we must determine what 20% of 200 is?

$$(.20)(200) = 40, \text{ Choice C}$$

9)   Answer: B   **Subject Area Tested: Percentages**

First, let's find the first discount amount: 30% of $80 = (.30)(80) = \$24.00$ discount. So now the price of the bicycle is $56 ($80 - $24 = $56).

Next, let's find the second discount amount: 30% of $56 = (.30)(56) = \$16.80$ discount. So now the bicycle is $39.20 ($56 - $16.80 = $39.20).

Finally, let's find the third discount amount: 25% of $39.20 = (.25)($39.20) = \$9.80$ discount. So now the bicycle is $29.40 ($39.20 - $9.80 = $29.40). Choice B is the correct answer.

10) Answer: B   **Subject Area Tested: Percentages**

Even though this problem tests your knowledge of percentages, we can solve this problem by using proportions and cross-multiplying. Set up the following proportion:

$$\frac{\text{the change from 1980 to 1981}}{\text{800 average in 1980}} = \frac{\text{unknown \%}}{\text{100\% of 1980's average}}$$

$$\frac{(1040 - 800)}{800} = \frac{X \%}{100\%}$$

$$\frac{240}{800} = \frac{X}{100}$$

$$24000 = 800 \ X$$

$$X = 30 \%, \text{ Choice B}$$

You may also solve the problem by asking: 240 (the difference) is what % of 800?
And then solve for the equation: $240 = 800 \ X$ ----------> $x = .3$, or 30%

11) Answer: B   **Subject Area Tested: Special Word Problems**
                                                     (Averages)

The problem tells us that the average of the five numbers is - 5. Therefore, the sum of the five numbers must be - 25 (the average is calculated by dividing the sum of the numbers by the quantity of the numbers). We are also told that the sum of three of the numbers is 15. 15 + the sum of the remaining two numbers must equal -25. Therefore the remaining two numbers must be - 40 (15 - (- 25) = 40). The average of the two remaining numbers is 20 (40 ÷ 2 = 20).
The answer is 20, Choice B.

12) Answer: E   **Subject Area Tested: Special Word Problems**
                                                 (Motion Problems)

To solve this motion problem, we must use the Distance Formula: Distance = Rate x Time

For the Atlanta to St. Louis trip: 1600 miles = 80 mph x T
                                         20 hours = Time

For the St. Louis to Atlanta trip: 1600 miles = 64 mph x T
                                          25 hours = Time
Total Time = 20 hours + 25 hours = 45 hours, Choice E

13) Answer: A   **Subject Area Tested: Special Word Problems**
                                                 (Motion Problems)

To solve this motion problem, we must use the Distance Formula: Distance = Rate x Time
Her overall Rate is affected by the current which is flowing against her. We are told she swam 8 miles in 2 hours. Let's find her actual Rate:

$$8 \text{ miles} = (R)(2 \text{ hours})$$
$$\text{Rate} = 4 \text{ miles per hour}$$

Since she was swimming at a rate of 6 mph, the current must have been flowing against her at a rate of 2 mph (6 mph - 4 mph = 2 mph). The answer is Choice A.

14) Answer: B   **Subject Area Tested: Special Word Problems**
                                                         (Ratios and Proportions)

We can quickly solve this problem by setting up the following proportion:

$$\frac{12 \text{ pounds of potatoes}}{8 \text{ pounds of potatoes}} = \frac{\$15.80}{\$ \, X}$$

If we get 12 pounds for $15.80, then we get 8 pounds for what? Cross multiply and solve for X:

$$12 \, X = \$126.40$$
$$X = \$10.53, \quad \text{Choice B}$$

15) Answer: A   **Subject Area Tested: Special Word Problems**
                                                         (Ratios and Proportions)

First , we must convert two years into months: 2 • 12 months = 24 months. Now, we can quickly solve this problem by setting up the following proportion:

$$\frac{5 \text{ months}}{24 \text{ months}} = \frac{48 \text{ models}}{X \text{ models}}$$

If she builds 45 models in 5 months, then she can build X models in 24 months. Cross multiply and solve for X:

$$5 \, X = 1152$$
$$X = 230.4, \text{ or 230 complete models, Choice A.}$$

16) Answer: C   **Subject Area Tested: Special Word Problems**
                                                         (Ratios and Proportions)

We can quickly solve this problem by setting up the following proportion:

$$\frac{30 \text{ prisoners}}{40 \text{ prisoners } (30 + 10 \text{ more})} = \frac{4 \text{ days}}{X \text{ days}}$$

If the food sustains 30 prisoners for 4 days, then it will sustain 40 prisoners for X days.

However,
you can not just cross multiply and solve for X. This is an INVERSE PROPORTION problem -- as you increase prisoners, the number of days of food supply goes down. So, we must first reverse (turn upside down) the second ratio before we cross multiply and solve for X.

$$\frac{30}{40} = \frac{X}{4}$$

$$40 \, X = 120$$

$$X = 3 \text{ days}, \text{ Choice C}$$

17) Answer: D   **Subject Area Tested: Special Word Problems**
                                                         (Work Problems)

To solve this problem, we must use the formula for Work Problems:

$$\frac{\text{Time Spent Working (1 Job)}}{\text{Time Needed to Complete Job}} = \text{Fractional Part of the Job or Rate}$$

AND: Fractional Part + Fractional Part = Whole Job

For Ryan alone: $\dfrac{1 \text{ Job (pruning shrubs)}}{4 \text{ Hours Needed to Complete Job}} = \dfrac{1}{4}$

For Ryan and Tyrone together:
$$\frac{1 \text{ Job}}{\quad} = \frac{9}{\quad}$$

$^{20}/_9$ Hours to Complete Job Together   20

So, now we know: Ryan's Part of the Work + the Unknown Part by Tyrone = Combined Effort

$$^1/_4 + ^1/_X = \; ^9/_{20}$$

Get rid of the fractions by multiplying through by a form of one $(^{20\,X}/_{20\,X})$ & solve for X:

$$5\,X + 20 = \; 9X$$

$$4\,X = \; 20$$

$$X = \; 5 \;\text{ hours, Choice D.}$$

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18) Answer: B   **Subject Area Tested: Special Word Problems**
                                            (Work Problems)

To solve this problem, we must use the formula for Work Problems:

$$\frac{\text{Time Spent Working (1 Job)}}{\text{Time Needed to Complete Job}} = \text{Fractional Part of the Job or Rate}$$

AND: Fractional Part + Fractional Part = Whole Job

For Beau alone: $\dfrac{1 \text{ Job}}{4 \text{ Hours Needed to Complete Job}} = \dfrac{1}{23 \text{ minutes}}$

For Nick alone: $\dfrac{1 \text{ Job}}{4 \text{ Hours Needed to Complete Job}} = \dfrac{1}{46 \text{ minutes } (2 \cdot 23)}$

Together: $^1/_{23} + ^1/_{46} = \; ^1/_X$

Remove fractions by multiplying through both sides with form of one $(^{46\,X}/_{46\,X})$ & solve for X:

$$2\,X + X = \; 46$$
$$3\,X = \; 46$$
$$X = \; 15\;^1/_3, \;\text{ Choice B.}$$

Note: you should realize that by working together, the boys can complete the job more quickly than either one of them working alone. Therefore, the answer must be less than 23 minutes, Beau's work time alone. So, you should quickly eliminate choices C, D, and E.

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19) Answer: B   **Subject Area Tested: Special Word Problems**
                                     (Unknown Integers Problems)

First, let's represent the smaller number by X. Since the larger number is "twice as large as the smaller number", we will represent it by 2X. The problem tells us that by increasing the smaller number by 12, (X + 12), we get five less than the larger number (2X - 5), or:

$$X + 12 = 2X - 5$$

Solve for X: $17 = X$

Solve for 2 X, the larger number: $2(17) = 34$, Choice B.

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20) Answer: C   **Subject Area Tested: Special Word Problems**
                                          (Interest Problems)

We can solve this problem by using Interest Problem formula:

$$(\text{Money Invested})(\text{Interest Rate \%}) = \text{Interest Income For 1 Year}$$

Plug in given information (if the quarterly rate is 4.5%, then the annual rate is (4 · 4.5 = 18%):

$$(\$40,000)(18\%) = \text{Interest Income for 1 Year}$$

$$\$7,200 = \text{Interest Income for 1 Year}$$

The problem ask for his interest income for 9 months, or $3/4$ of a year. Therefore, we must take $3/4$ of his annual interest revenue to get the correct answer.

$$(^3/_4)\ (\$7,200)\ =\ \$5,400,\ \text{Choice C.}$$

21) Answer: A   **Subject Area Tested: Special Word Problems
(Symbol Representation Problems)**

These problems are easy to solve if you take an organized approach. First, substitute the given values for the symbols into the given definition (equation) of the symbols. The given values are 4, 3, and v. So, as in the symbols' definition, where we see a y in the given equation, we now substitute a 4. We replace all of the o's with 3's and all z's with v's: y = 4, o = 3, and z = v.

So: oy + oz = 0 becomes:

$$(3)\ (4)\ +\ (3)\ (v)\ =\ 0$$
$$12\ +\ 3v\ -=\ 0,\ \text{therefore,}\ 3v\ =\ -12\ \text{and}$$
$$v\ =\ -4,\ \text{Choice A.}$$

22) Answer: C   **Subject Area Tested: Plane Geometry**

First, we should draw a picture:

Now, by using Pythagoreans Theorem, we can solve for the missing leg, b, the distance between town and Carlo's house.

$$\text{leg}^2\ +\ \text{leg}^2\ =\ \text{hypotenuse}^2$$
$$(16)^2\ +\ X^2\ =\ [(16+X)-8]^2$$
$$256\ +\ X^2\ =\ [8+X]^2$$
$$256\ +\ X^2\ =\ 64\ +\ 16X\ +\ X^2$$
$$192\ =\ 16X$$
$$X\ =\ 12,\ \text{Choice C.}$$

23) Answer: E   **Subject Area Tested: Plane Geometry**

The third side of the triangle has two possible lengths, depending upon whether the third side is a leg or the hypotenuse. Let's use the Pythagorean Theorem and find both of these possible values:

First scenario: Right triangle with leg = 1, leg = X, unknown third side, and hypotenuse = $\sqrt{2}$:

$$\text{leg}^2\ +\ \text{leg}^2\ =\ \text{hypotenuse}^2$$
$$(1)^2\ +\ X^2\ =\ (\sqrt{2})^2$$
$$X^2\ =\ 2\ -\ 1$$
$$X\ =\ \sqrt{1},\ \text{or 1}$$

Second scenario: Right triangle with leg = 1, leg = $\sqrt{2}$, and hypotenuse = X, unknown side:

$$\text{leg}^2 + \text{leg}^2 = \text{hypotenuse}^2$$

$$(1)^2 + (\sqrt{2})^2 = X^2$$
$$1 + 3 = X^2$$
$$X = \sqrt{3}$$

The length of the third side could be 1, or I and III, Choice E.

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24) Answer: D   **Subject Area Tested: Geometry**

We know that a cube has 6 sides and that each of theses 6 sides is an identical square. The surface area of a cube is the sum of the areas of these six squares. We are told that the surface area of the cube is 96 square inches. Let's first find the length of the squares' sides.

$$6 * \text{Area of one square} = \text{Surface Area of Cube}$$
$$(6)\ S^2 = 96$$
$$S^2 = 16$$
$$S = 4$$
$$\text{Volume of the Cube} = S^3$$

Therefore, $\text{Volume of the Cube} = (4)^3 = 64$, Choice D

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25) Answer: E   **Subject Area Tested: Coordinate Geometry**

To determine which pair of coordinates lies on the circle, you first must realize that the radius of the circle is 15. The coordinates (15, 0) on the drawing shows us that from the center of the circle to the edge of the circle is 15 units. The Pythagorean theorem states that $a^2 + b^2 = c^2$. For a pair of coordinates to lie on the circle, the hypotenuse, or the c value in the equation, must equal 15, the radius.

Each pair of coordinates forms a right triangle on the graph. For instance, (-12, 13) dictates that you should go left from the origin 12 units on the X axis, then turn ninety degree up and go 13 units. You can solve for the hypotenuse (which is the distance from the coordinate point directly to the Origin) by using the Pythagorean theorem. The pair of coordinates provided in the answer choices which yields a hypotenuse of 15, or a c^2 value of 225 (15 x 15), is the correct answer.

Try each of the answer choices:

(A) $(-12)^2 + (13)^2 = 144 + 169 = 313 \neq 225$ WRONG

(B) $(-12)^2 + (12)^2 = 144 + 144 = 288 \neq 225$ WRONG

(C) $(13)^2 + (-11)^2 = 169 + 121 = 290 \neq 225$ WRONG

(D) $(13)^2 + (9)^2 = 169 + 81 = 250 \neq 225$ WRONG

(E) $(12)^2 + (9)^2 = 144 + 81 = 225 = 225$ **CORRECT**

The coordinates of Choice E, (12, 9), produce an hypotenuse of 15, which lies on the circle.