

1. In the figure below, the spinner has five equal sections numbered 1 through 5. If the arrow is equally likely to land on any of the sections, what is the probability that it will land on an even number on the next spin?



Since there are two even numbers (2 and 4), the probability is $\frac{2}{5}$ that an even number will be landed on during any spin.

The correct answer is B, $\frac{2}{5}$.

2. Which of the following is the equation of a line parallel to the line $y - 2 = 3x$?

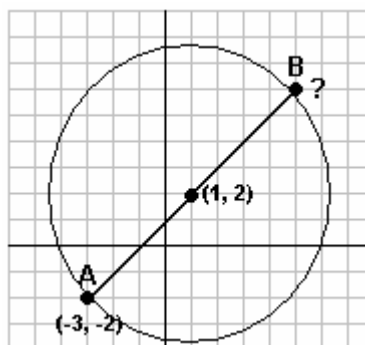
Since the answers are in $y = mx + b$ form, rewrite given equation in this form:

$$\begin{aligned} y - 2 &= 3x \\ y &= 3x + 2 \end{aligned}$$

The slope of the line is 3, and a parallel line has the same slope.

The correct answer is H, $y = 3x + 1$.

3. Circle O has center $(1, 2)$ and diameter AB . If the coordinates of A are $(-3, -2)$, what are the coordinates of B ?



First sketch a graph including points $(1, 2)$ and $(-3, -2)$. From point A to the center of the circle, one would go up 4 units and to the right 4 units. Following the same path from the center of the circle to point B, from $(1, 2)$ up 4 and to the right 4 would end at the point $(5, 6)$.

The correct answer is C, $(5, 6)$.

4. If $x = -2$, then $-x^2 - 2x - 3 = ?$

Substitute -2 for each x in the expression, using parentheses for each substitution.

$$\begin{aligned} & -x^2 - 2x - 3 \\ & -(-2)^2 - 2(-2) - 3 \\ & -4 + 4 - 3 \\ & -3 \end{aligned}$$

The correct answer is K, -3 .

5. What is the solution set, in terms of r , of the following system of equations?

$$\begin{aligned} 3x + y &= 2r \\ x + y &= r \end{aligned}$$

This system of equations is best solved using the addition method (sometimes called the elimination method). First, multiply the second equation through by -1 . Then add the two equations.

$$\begin{aligned} 3x + y &= 2r \\ (-1)(x + y) &= r(-1) \end{aligned}$$

becomes

$$\begin{aligned} 3x + y &= 2r \\ -x + (-y) &= -r \end{aligned}$$

Now add the two equations

$$\begin{array}{r} 3x + y = 2r \\ -x + (-y) = -r \\ \hline 2x \quad = r \\ x = \frac{r}{2} \end{array}$$

We find that $x = \frac{r}{2}$. Then substitute this back into the second equation (the

simpler of the two) and we find that $y = \frac{r}{2}$.

$$\frac{r}{2} + y = r$$

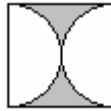
$$r - \frac{r}{2} = y$$

$$\frac{r}{2} = y \quad (\text{because } r$$

minus one-half r is equal to one-half r)

The correct answer is B, $x = \frac{r}{2}$ and $y = \frac{r}{2}$, written in (x,y) form $(\frac{r}{2}, \frac{r}{2})$.

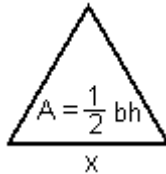
6. In the figure below, two tangent semicircles are drawn in a square. If the length of a side of the square is 4, what is the area of the shaded portion of the figure?



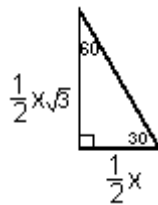
The area of the square is 16 ($A = s^2$). Since the side of the square is 4, the radius of the circle is 2 and the area of the circle (both halves equal one whole circle) is $4\pi r^2$.

The shaded area is therefore $16 - 4\pi r^2$. The correct answer is J.

7. The area of an equilateral triangle is $\frac{25\sqrt{3}}{4}$. What is the perimeter of the triangle?



Let the base be x and find the height. Since this is an equilateral triangle, each angle is 60° . Half of this triangle is a 30-60-90 triangle. In this type of triangle, the middle length (the height) is the short length times $\sqrt{3}$. Therefore the height is $\frac{1}{2}\sqrt{3}x$.



Now, use the area formula (with original triangle):

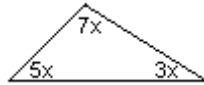
$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(x)\left(\frac{1}{2}\sqrt{3}x\right)$$

$$A = \frac{x^2\sqrt{3}}{4}$$

Since the area is $\frac{25\sqrt{3}}{4}$, x must equal 5. Therefore, the perimeter is 15. The correct answer is C, 15.

8. The measures of the angles of a triangle are in the ratio of 3 : 5 : 7. What is the measure in degrees, of the smallest angle ?



Assign angles of a triangle $3x$, $5x$ and $7x$. Since all angles add to 180 degrees:

$$3x + 5x + 7x = 180$$

$$15x = 180$$

$$x = 12$$

So the angles are 36° ($3x$), 60° ($5x$) and 84° ($7x$). The correct answer is the smallest angle 36° , H.

9. The expression $(x - y)(x^2 + xy + y^2)$ is equivalent to which of the following?

Multiply (distribute) as follows: $x(x^2 + xy + y^2) - y(x^2 + xy + y^2)$

$$x^3 + x^2y + xy^2 - x^2y - xy^2 - y^3$$

Combining like terms, the result is $x^3 - y^3$. The correct answer is C, $x^3 - y^3$.

10. What is the value of $(2a^2 - a^3)^2$ when $a = -1$?

Substitute -1 for each a in the expression, using parentheses for each substitution.

$$(2(-1)^2 - (-1)^3)^2$$

$$(2(1) - (-1))^2$$

$$(2 + 1)^2$$

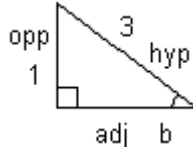
$$(3)^2$$

$$9$$

The correct answer is J, 9.

11. If $\sin x = \frac{1}{3}$, which of the following could be a value of $\tan x$?

Since $\sin x = \frac{\text{opposite}}{\text{hypotenuse}}$, then the following right triangle applies:



Use Pythagorean Theorem to find missing side, b.

$$b^2 + 1^2 = 3^2$$

$$b^2 + 1 = 9$$

$$b^2 = 8$$

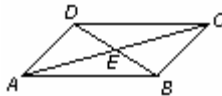
$$b = \sqrt{8}$$

Since $\tan x = \frac{\text{opposite}}{\text{adjacent}}$, $\tan x = \frac{1}{\sqrt{8}}$ or rationalizing the denominator, $\frac{1}{\sqrt{8}} \cdot \frac{\sqrt{8}}{\sqrt{8}} =$

$$\frac{\sqrt{8}}{8} = \frac{2\sqrt{2}}{8} = \frac{\sqrt{2}}{4}$$

The correct answer is B, $\frac{\sqrt{2}}{4}$.

12. In the figure below, ABCD is a parallelogram. If $EC = 31$, $EB = 27$ and $AE = 4x - 5$, then $x = ?$



Since ABCD is a parallelogram, $(AE) = (EC)$

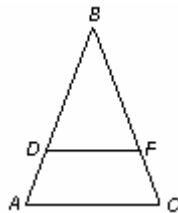
$$31 = 4x - 5$$

$$36 = 4x$$

$$x = 9$$

The correct answer is H, 9.

13. In the diagram below, $\triangle ABC$ is similar to $\triangle DBF$. If $DF = 3$, $BD = BF = 6$, and $AC = 4$, what is the perimeter of $\triangle ABC$?



Since $\triangle ABC$ is similar to $\triangle DBF$ and $DF = 3$ and $AC = 4$, the ratio of sides is 3:4. Then since $BD = BF = 6$, AB and BC must equal 8. Therefore the perimeter of

$\triangle ABC = 4 + 8 + 8 = 20$. The correct answer is C, 20.

14. Two roads intersect at right angles. A pole is 30 meters from one road and also 40 meters from the other road. How far (in meters) is the pole from the point where the roads intersect?

The roads form a triangle with sides 30m and 40m. Since it is a right triangle, we know that the missing side is 50m. This is a 3-4-5 right triangle). Or using Pythagorean theorem,

$$30^2 + 40^2 = c^2$$

$$900 + 1600 = c^2$$

$$2500 = c^2$$

$$50 = c$$

The correct answer is H, 50.

15. From the town of Williston Park to Albertson there are 3 different roads. From the town of Albertson to Mineola there are 5 routes. How many different paths are there to go from Williston Park to Mineola through Albertson?

Since there are 3 roads to Albertson and then 5 roads to Mineola, there are a total of 15 distinct paths.

The correct answer is E, 15.

16. What is the y-intercept of the line whose equation is $(y - 5) = \frac{1}{2}(x + 2)$?

This equation is in point-slope form. Rewrite to slope-intercept form to determine the y-intercept.

$$(y - 5) = \frac{1}{2}(x + 2)$$

$$y - 5 = \frac{1}{2}x + 1$$

$$y = \frac{1}{2}x + 6$$

The y-intercept is 6.

The correct answer is H, 6.

17. For which value of c will the roots of the equation $x^2 + 4x + c = 0$ be real and equal?

Factor by using reverse FOIL assuming that the middle term $4x$ must be the sum of $2x + 2x$.

$$(x + 2)(x + 2) = 0$$

In this case $x = -2$. Therefore the roots are real and equal. Using FOIL to arrive at the original equation, $x^2 + 4x + 4 = 0$

Therefore $c = 4$.

The correct answer is D, 4.

18. Given the true statement, "If it is raining, then the sky is green." Which of the following is also a true statement?

Use a simpler example such as "If you are 17, then you are a teenager." Rewrite each choice substituting **17** for raining and **teenager** for sky is green.

- F. If the sky is green, then it is raining.
- F. If you are a teenager, then you are 17. **FALSE**
- G. If it is not raining, the sky is not green.
- G. If you are not 17, then you are not a teenager. **FALSE**
- H. If the sky is not green, then it is not raining.
- H. If you are not a teenager, then you are not 17. **TRUE**
- J. If the sky is not green, then it is raining.
- J. If you are not a teenager, then you are 17. **FALSE**
- K. If it is not raining, then the sky is blue.
- K. If you are not 17, then you are a preschooler. **FALSE**

The correct answer is H.

19. If $f(x) = 2x - 5$ and $g(x) = \frac{x + 5}{2}$ what is the value, in terms of m , of $f(g(2m))$?

First substitute $2m$ into $g(x)$.

$$g(2m) = \frac{2m + 5}{2}$$

Now, substitute $\frac{2m + 5}{2}$ into $f(x)$.

$$f(g(2m)) = 2\left(\frac{2m + 5}{2}\right) - 5$$

$$f(g(2m)) = 2m + 5 - 5$$

$$f(g(2m)) = 2m$$

The correct answer is B, $2m$.

20. Which graph represents the solution set of the inequality $(x - 1)(x + 3) < 0$?

Set each factor equal to zero.

$$(x - 1) = 0 \quad x = 1$$

$$(x + 3) = 0 \quad x = -3$$

These are the points marked with open circles (since $<$ is used). So it must be F or G. Now test $x = 0$ in original inequality.

$$(0 - 1)(0 + 3) < 0?$$

$$(-1)(3) < 0?$$

$$(-3) < 0 \text{ TRUE}$$

Since we have a true statement when $x = 0$, then 0 must be in the solution set, so the correct answer is F.

ACT Math Diagnostic Test 2

Answer Key

