

1. Matrix Multiplication

Given $A = \begin{bmatrix} 3 & 1 & -1 \\ 2 & 4 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ -1 & 3 \\ 0 & 4 \end{bmatrix}$ find the sum of the elements of the product BA .

First, rewrite the matrices in the correct order to multiply.

$B = \begin{bmatrix} 1 & 2 \\ -1 & 3 \\ 0 & 4 \end{bmatrix}$ The product is BA has order 3×3 since B is sized 3×2 and A is sized 2×3 .

$A = \begin{bmatrix} 3 & 1 & -1 \\ 2 & 4 & 5 \end{bmatrix}$

$BA = \begin{bmatrix} 7 & 9 & 9 \\ 3 & 11 & 16 \\ 8 & 16 & 20 \end{bmatrix}$. Now, add the sum of the elements to get choice E, 99.

2. Matrix Multiplication

Which matrix product is undefined ?

$M = \begin{bmatrix} 1 & -3 \\ 2 & 5 \end{bmatrix}$ $N = \begin{bmatrix} 2 & 4 \\ -3 & 1 \end{bmatrix}$ $P = \begin{bmatrix} 1 & 3 & -7 \\ 4 & 2 & 3 \end{bmatrix}$ $R = \begin{bmatrix} 5 & 7 \\ 8 & 1 \\ 0 & 7 \end{bmatrix}$

M and N are both order 2×2 , so MN and NM can both be found.

P is order 2×3 and R is order 3×2 , so PR and RP can both be found.

MP can also be found, but since N is 2×2 and R is 3×2 , NR is undefined. The correct choice is K, NR .

3. Equation of an Ellipse

Find the focus points for the ellipse $\frac{x^2}{6} + \frac{y^2}{10} = 1$ using the formula $a^2 = b^2 + c^2$.

Since $a^2 > b^2$ for ellipses, this ellipse has a vertical major axis with the equation $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$.

Now calculate $a^2 = b^2 + c^2 \rightarrow 10 = 6 + c^2 \rightarrow c^2 = 4$, with $c = \pm 2$.

Since the ellipse has a vertical major axis, the foci are on the y -axis with coordinates $(0, 2)$ and $(0, -2)$, which is choice B.

4. Equation of a Parabola

Find the distance between the focus of the parabola $f(x) = -2(x-3)^2 + 1$ and its directrix using the formula $a = \frac{1}{4p}$.

Comparing the given equation to $y = \frac{1}{4p}(x-h)^2 + k$, we see that $-2 = \frac{1}{4p}$ so $p = \frac{-1}{8}$. The distance between the focus and the vertex is therefore $|p| = \frac{1}{8}$.

This must be doubled to find the distance between the focus and the directrix, so the correct choice is J, $\frac{1}{4}$.

5. Equation of a Hyperbola

What are the slopes of the asymptotes for the hyperbola $\frac{(y-3)^2}{9} - \frac{(x-2)^2}{16} = 1$?

With formula $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$, we see that $a = 3$ and $b = 4$.

Asymptotes for a hyperbola with a vertical transverse axis are found by using $y = \pm \frac{a}{b}x$, with slopes $\pm \frac{a}{b}$.

Since a^2 is paired with y^2 and b^2 is paired with x^2 , $a = 3$ represents the vertical change and $b = 4$ represents the horizontal change of the hyperbola's fundamental rectangle.

The correct choice is C, $\pm \frac{3}{4}$.

6. Arithmetic Sequences

A cross country team runs one half of a mile on the first day of practice and increases the distance by the same amount each practice session. If 8 miles are run on the 31st day of practice, how much was the distance increased each day?

Use the formula $a_n = a_1 + (n-1)d \rightarrow 8 = \frac{1}{2} + (31-1)d \rightarrow d = \frac{1}{4}$.

The correct choice is H, $\frac{1}{4}$ mile.

7. Geometric Series

If a ball is dropped from a height of 50 feet and it always rebounds 0.8 of the distance that it falls, approximately how far does it travel before coming to rest?

The bouncing ball represents an infinite geometric series. Use the formula $S = \frac{a}{1-r}$ to find the total distance traveled.

The series is represented by $50 + 40 + 40 + 32 + 32 + 25.6 + 25.6 + \dots$

Use the formula with $a = 40$ and $r = 0.8$ to find the rebound total distance only. Then double this sum and add 50 to find the total sum.

$S = \frac{a}{1-r} \rightarrow S = \frac{40}{1-0.8} \rightarrow S = 200$. Now add $200 + 200 + 50 = 450$ feet.

The correct choice is E, 450 feet.

8. Asymptotes of a Rational Function

Find the equations of all asymptotes for $f(x) = \frac{3x^2 + 7x + 2}{x^2 - 4}$

This rational function factors as $f(x) = \frac{(3x+1)(x+2)}{(x-2)(x+2)}$ and reduces to $f(x) = \frac{3x+1}{x-2}$ with a hole when $x = -2$.

The vertical asymptote is $x = 2$ and the horizontal asymptote is $y = 3$.

The correct choice is G, $x = 2$, $y = 3$, hole when $x = -2$.

9. Conditional Probability

Find the probability that a student plays in marching band, given that the student performed in the spring play.

Student Activities	Plays in Marching Band	Plays in Orchestra	Does not Play an Instrument
In the Spring Play	14	15	10
In the Fall Play	16	20	12
Not in a Play	20	10	85

By using the word "given," this is a conditional probability question. The sample space consists of the 39 students who performed in the spring play. Of these, 14 students play in marching band.

The correct choice is D, $\frac{14}{39}$.

10. Permutations

If 4 students are randomly selected from a class of 14 to serve as president, vice-president, treasurer, and secretary, what is the probability that the 4 youngest students in the class are selected?

This is a permutation question since the order matters for the 4 students who serve as president, vice-president, treasurer, and secretary. Calculating ${}_{14}P_4$ gives 24,024 permutations. Of these, only one permutation includes the four youngest students in the class.

The correct choice is F, $\frac{1}{24024}$.

11. Combinations

If a local pizza restaurant has 17 toppings available, how many different pizzas are available if the customer can choose any number of different toppings between 0 and 17? Assume that the order of the toppings does not matter.

This is a combination question since the order of the toppings does not matter. The long way to solve this problem is adding 18 different combination calculations beginning with ${}_{17}C_0$ and ending with ${}_{17}C_{17}$. However, since this is a binomial situation, each topping is either included or not included. This gives 2 choices for each of the 17 toppings.

Therefore, simply calculate $2^{17} = 131,072$.

The correct choice is D, 131072.

12. Amplitude of a Trigonometric Function

What is the amplitude of the function $f(x) = \frac{1}{4} \sin(3x - \pi)$?

Using the equation $f(x) = a \sin(bx - c) + d$, the amplitude is $a = \frac{1}{4}$.

The correct choice is F, $\frac{1}{4}$.

13. Vectors

What is the magnitude of the vector $\mathbf{b} = (6, 8)$?

Use the Pythagorean Theorem to find the magnitude.

$$\text{Magnitude} = \sqrt{6^2 + 8^2} = \sqrt{100} = 10$$

The correct choice is C, 10.

14. Terminal Sides

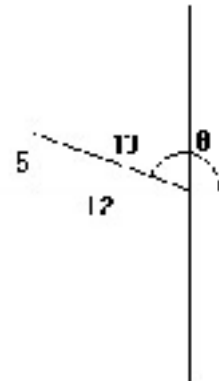
If $(-12, 5)$ is a point on the terminal side of the angle θ , find $\cos \theta$.

A reference angle is the smallest angle that the terminal side of a given angle makes with the x-axis. In Quadrant II, the reference angle is $180 - \theta$.

Consider the $(5 -12 -13)$ reference right triangle in quadrant II, where $\sin \theta > 0$, $\cos \theta < 0$, and $\tan \theta < 0$.

Since $\cos \theta = \frac{\text{adj}}{\text{hyp}}$, this gives $\cos \theta = -\frac{12}{13}$.

The correct choice is H, $-\frac{12}{13}$.



15. Coterminal Angles

What is the measure of the smallest negative angle that is coterminal with 110 degrees?

To find a negative angle coterminal with a given angle, subtract 360° .

$$110^\circ - 360^\circ = -250^\circ$$

The correct choice is A, -250° .

16. Best Fit Line

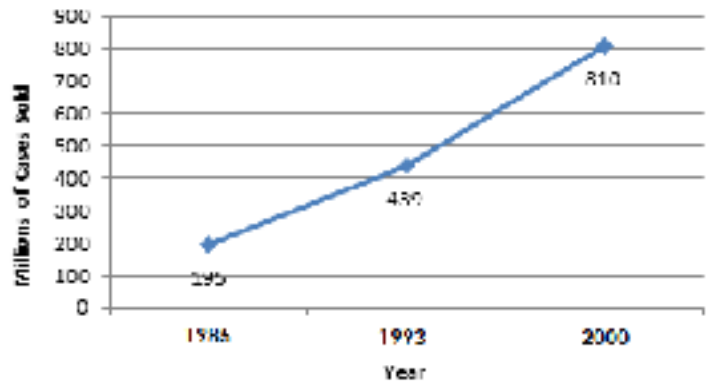
Use the graphed data points to predict the number of cases sold for a popular soft drink in the year 2017.

The line between 1993 and 2000 is the better predictor, since it gives more recent sales data.

The slope of that line is $m = \frac{810 - 439}{2000 - 1993} = \frac{371}{7}$

If the year 2000 is considered year 0, allowing $x = 17$ for year 2017 yields $\frac{371}{7} \times 17 = 901$. Add this to the starting point of 810 million cases in year 2000 for a total of 1711 million cases.

The correct choice is J, 1711 million cases.



17. Modeling

If 537^{102} were calculated, it would have 279 digits. What would the digit farthest to the right be (the ones digit)?

Start by noticing the pattern:

Calculating 537^1 ends in 7

Calculating 537^2 ends in 9

Calculating 537^3 ends in 3

Calculating 537^4 ends in 1

Calculating 537^5 ends in 7

Since 7 – 9 – 3 – 1 keeps repeating in groups of four, 537^{100} would end in 1 and 537^{102} would end in 9.

The correct choice is E, 9.

18. Domain of a Function

Find the domain of the inverse function for $f(x) = \sqrt{x-3}$.

The range of the original function becomes the domain of the inverse of that function.

Since the original function has range $[0, \infty)$, the domain of the inverse of that function is also $[0, \infty)$.

If the inverse is calculated, it would be $f^{-1}(x) = x^2 + 3$ but only when $x \geq 0$, which is written in interval notation as $[0, \infty)$.

The correct choice is J, $[0, \infty)$.